Name: ____________________ Section: ____________________

## Taking Derivatives

1. Compute the derivative of $f(x) = \sin(x^2 + x + 1)$

2. Compute the derivative of $f(x) = \cos(x^2) \cdot \sin(x^2)$

3. Compute the derivative of $f(x) = \sin(x \cdot e^x)$

4. Compute the derivative of $f(x) = \sin(e^{x^2+x+1})$

Name: ____________________ Section: ____________________

5. Suppose that $x \cdot y = x^2$. Find $\dfrac{dy}{dx}$.

6. Suppose that $y^2 + 2xy + x^2 = 1$. Find $\dfrac{dy}{dx}$.

7. Suppose that $e^y = \cos(x)$. Find $\dfrac{dy}{dx}$.

8. Suppose that $e^y = \cos(y) + x$. Find $\dfrac{dy}{dx}$.

Name: ______________________ Section: ______________________

When taking the derivative of a function containing logarithms, try using the 3 laws of logarithms to simplify first.

9. Let $f(x) = \ln\left(\sqrt{\dfrac{x^2+4x+4}{x^2-1}}\right)$. Find $f'(x)$

10. Let $f(x) = \ln\left(x^{\sin(x)}\right)$. Find $f'(x)$

Name: ____________________ Section: ____________________

Introduce logarithms when computing certain complicated derivatives

11. Let $f(x) = \sqrt{\dfrac{x^2+4x+4}{x^2-1}}$. Find $f'(x)$.

12. Let $f(x) = \Big(\ln(x)\Big)^{\sin(x)}$. Find $f'(x)$.

Name: ____________________ Section: ____________________

## Concrete Applications of Derivatives

1. Suppose that a population of bacteria is growing in a petri dish. Suppose also that the first time you look at the dish you count 20 bacteria, and that you count 200 bacteria in the dish 3 hours later. Find a formula for the population as a function of the number of hours $t$ since your first measurement.

   How much time is required for the population to double in size?

2. Suppose that Ebola is spreading through the city of Waterbury. Four people are ill two days into the outbreak, and eight people are ill four days in. Find a formula for the number ill a function of days since the outbreak began (0 days in).

   How long until 100 people are ill?

3. Suppose that you begin with 100 grams of a radioactive substance. Suppose also that the substance has a half life of 3 years. Find a formula for the amount of radioactive substance remaining after $t$ years.

   What is the weight of the radioactive substance that remains after 9 years?

Name: ____________________ Section: ____________________

4. An airplane flies directly over a radar station, at a constant altitude of 3 mi above the ground. A little while later, the radar station measures that (a) the distance between the plane and radar station equals 5 mi and (b) that the distance between the plane and radar station is increasing at a rate of 500 mi/hr. What is the ground speed of the airplane at the time of the second measurement?

5. An ice cube melts, with its surface area decreasing at a rate of 3 $\text{in}^2$/s. How fast is the side length decreasing when the side length is 1 in?

6. A streetlight is mounted at the top of a 6 meter pole, and a 2 meter tall person is walking toward it at 2 meters per second. How fast is the length of their shadow changing when they are 4 meters from the streetlight? What about when they are 1 meter from the light?

7. A police officer is walking down a city street, when they spot a wanted felon standing 200 ft away at the corner of the next block. The police officer takes off after the felon at 12 ft/s, and the felon immediately cuts around the corner and runs away at 9 ft/s. What is the rate of change of the distance between the officer and the felon after 10 seconds have passed?

8. Suppose there is a 100 cm long water trough which is empty at time $t = 0$. The cross-section of the trough is an inverted triangle $\nabla$ which is 20 cm across the top, and is 10 cm tall. If the tank is being being filled with water at a constant rate of 400 $\text{cm}^3$/s, how fast is the height changing when the tank is half full?

9. Suppose the water trough above leaks (100 cm long, cross section is a $\nabla$, top $=$ 20 cm, and height $=$ 1 cm). If water is being added to the tank at a rate of 400 $\text{cm}^3$/s, and is leaking out of the tank at 100 $\text{cm}^3$/s, how fast is the height changing when the tank is half full?

Name: ____________________ Section: ____________________

## Abstract Applications of Derivatives

1. Find a linear approximation for the function $f(x) = \sin(x)$ at $a = \frac{\pi}{4}$.

   Use your answer to approximate $\sin\left(\frac{5\pi}{16}\right)$.

2. Find a linear approximation for the function $f(x) = \cos(x)$ at $a = \frac{\pi}{4}$

   Use your answer to approximate $\cos\left(\frac{5\pi}{16}\right)$.

3. Find a linear approximation for the function $f(x) = \tan(x)$ at $a = \frac{\pi}{4}$

   Use your answer to approximate $\tan\left(\frac{5\pi}{16}\right)$.

Name: ____________________ Section: ____________________

4. Let $f(x) = x^3 - 6x^2 + 9x + 1$

   Find the following. If a requested quantity doesn't exist, answer "DNE".

   (a) The intervals where $f(x)$ is increasing/decreasing. Identify which is which.

   (b) The intervals where $f(x)$ is concave up/down. Identify which is which.

   (c) The $x$ value(s) of the local maxima and local minima of $f$. Identify which is which.

   (d) The $x$ value(s) of the inflection points of $f$.

Name: ______________________ Section: ______________________

Use L'Hospital's Rule to answer the following limits. Remember to **show all work**.

This is the best way to learn to do these problems correctly!

5. Does the limit $\lim_{x\to 0+} \dfrac{\ln(x^3)}{x^3+3}$ converge? If so, what does it converge to?

6. Does the limit $\lim_{x\to \infty} \dfrac{\ln(x^3)}{x^3+3}$ converge? If so, what does it converge to?

7. Does the limit $\lim_{x\to \infty} 2x\sin\left(\dfrac{1}{2x}\right)$ converge? If so, what does it converge to?

8. Does the limit $\lim_{x\to \infty} \left(1+\dfrac{8}{x}\right)^x$ converge? If so, what does it converge to?

Name: ______________________ Section: ______________________

9. Find the antiderivative of $f(x) = \sin(x)$.

10. Find the antiderivative of $f(x) = \cos(x)$.

11. Find the antiderivative of $f(x) = \sec^2(x)$.

12. Find the antiderivative of $f(x) = \dfrac{1}{\sqrt{x}}$.

13. Find the antiderivative of $f(x) = \dfrac{1}{x}$.

14. Find the antiderivative of $f(x) = \dfrac{1}{x^2}$.

15. Find the antiderivative of $f(x) = \dfrac{1}{x^3}$.

16. Suppose $f'(x) = x^2 + 2x + 5$ and that $f(0) = 0$. Find $f(x)$.

17. Suppose $f'(x) = \frac{1}{x} + x + e^x$ and that $f(1) = 0$. Find $f(x)$.